Derivatives
Multiple Choice & Free Response

1. $y^3 - xy^2 = 4$

$3y^2\frac{dy}{dx} - [(1)y^2 + (x)(2y)\frac{dy}{dx}] = 0$

$3y^2\frac{dy}{dx} - y^2 - 2xy\frac{dy}{dx} = 0$

$3y^2\frac{dy}{dx} - 2xy\frac{dy}{dx} = y^2$

$\frac{dy}{dx} = \frac{y^2}{3y^2 - 2xy}$

$(2)^3 - x(2)^2 = 4$

$8 - 4x = 4$

$-4x = -4$

$x = 1$

$m_{tan} = \frac{(2)^2}{3(2)^2 - 2(1)(2)} = \frac{4}{12-4} = \frac{4}{8} = \frac{1}{2}$

D

2. $y^2 - xy - 3x = 1$

$2y\frac{dy}{dx} - [(1)y + (x)\frac{dy}{dx}] - 3 = 0$

$2y\frac{dy}{dx} - y - x\frac{dy}{dx} - 3 = 0$

$2y\frac{dy}{dx} - x\frac{dy}{dx} = y + 3$

$\frac{dy}{dx} = \frac{y+3}{2y-x}$

$m_{tan} = \frac{(-1)+3}{2(-1)-(0)} = \frac{2}{-2} = -1$

A

3. $y = x\sin x$

$y' = (1)\sin x + (x)\cos(x)$

$m_{tan} = \sin(\frac{\pi}{2}) + (\frac{\pi}{2})\cos(\frac{\pi}{2})$

$m_{tan} = (1) + (\frac{\pi}{2})(0)$

$m_{tan} = 1$

$y - \frac{\pi}{2} = 1(x - \frac{\pi}{2})$

$y = x$

E

4. $y = xe^{-x}$

$y' = (1)e^{-x} + (x)(e^{-x})(-1)$

$y' = e^{-x} - xe^{-x}$

$0 = e^{-x}(1-x)$

$x = 1$ **B1**

5. $y = (2x+1)^{1/2}$ $\quad y = -3x + 6$

$y' = \frac{1}{2}(2x+1)^{-1/2}(2)$ $\quad m = -3$

$\frac{1}{3} = \frac{1}{\sqrt{2x+1}}$ $\quad m_{\perp} = \frac{1}{3}$

$(\sqrt{2x+1})^2 = (3)^2$ $\quad \sqrt{2(4)+1} = 3$

$2x + 1 = 9$

$2x = 8$

$x = 4$ $\quad (4, 3)$ **A1**

6. $y^2 - xy + 9 = 0$

$2y\frac{dy}{dx} - [(1)y + (x)\frac{dy}{dx}] = 0$

$2y\frac{dy}{dx} - y - x\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} - x\frac{dy}{dx} = y$ $\quad y^2 - (2y)y + 9 = 0$

$\frac{dy}{dx} = \frac{y}{2y-x}$ $\quad y^2 - 2y^2 + 9 = 0$

$-y^2 = -9$

$0 = \frac{2y-x}{y}$ $\quad y^2 = 9$

$y = \pm 3$

$0 = 2y - x$

$x = 2y$

D1

7. $V = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dv}{dt} = 4\pi(3)^3(.1)$

$\frac{dv}{dt} = 33.9$ **D**

8. $xy = 2$

$[(1)y + (x)\frac{dy}{dx}] = 0$

$y + x\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-y}{x}$

$x^2 - y^2 = 3$

$2x - 2y\frac{dy}{dx} = 0$

$2x = 2y\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{x}{y}$

* orthoganal at all values of x & y

$\therefore m_1 m_2 = -1$ **B**

9. $y = 3x + k$

$m_{tan} = 3$

$y = x^3$

$y' = 3x^2$

$3 = 3x^2$

$1 = x^2$

$x = \pm 1$

$(-1, -1)$

$(1, 1)$

$-1 = 3(-1) + k$

$-1 = -3 + k$

$k = 2$

$1 = 3(1) + k$

$-2 = k$

E

10. $s = t^4 - 6t^3 + 12t^2 + 3$

$v = 4t^3 - 18t^2 + 24t$

$0 = 2t(2t^2 - 9t + 12)$

$0 = 2t(2t \quad \times t \quad)$

$t = 0$ **B**

11. $s = t^5 + 5t^4$

$v = 5t^4 + 20t^3$

$0 = 5t^3(t + 4)$

$t = 0 \quad t = -4$ **A**